Prevention efforts and insurance demand under coherent risk measures

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Motivations

Study of actions taken by agents to **reduce their risk exposure**.

1. Transfer the risk to a counterparty (Well studied by Nicole: risk transfer, weather derivatives, inf-convolution of risk measures...)
    - Insurance
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    - Insurance linked securities (Cat bonds...)

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   - Reinsurance
   - Insurance linked securities (Cat bonds...)
2. Take prevention actions.
   - Optimal choice between market insurance and prevention activities.
   - Self-protection: reduction of the **probability** to suffer a claim.
   - Self-insurance: reduction of the claim **amount**.

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- market insurance and self-protection could be **complements**, depending on the level of the loss probability.
- Led to many discussions and extensions on the optimal individual behaviour with respect to prevention (Bleichrodt, Briys, Chiu, Courbage, Dionne, Eeckhoudt, Gollier, Konrad, Rey, Schlesinger, Skaperdas, Treich etc.)

A model inspired by contract theory

Principal-agent type model where:

- The agent $=$ insurance buyer whose action is given by (α, e) with
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  $\pi(X) := (1+\theta)\mathbb{E}[X]$, with $\theta \geq 0$.
- Losses = r.v. $(X_e)_{e \in (0,+\infty)}$ whose distributions form a family of probability measures which is decreasing for the first order stochastic dominance.

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where ρ_A is a given law invariant coherent risk measure, and c is a non-decreasing convex function.

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where ρ_P is a given law invariant coherent risk measure, and $\mathcal{A}$ is the set of prices accepted by the buyer.

Some results

Main Assumption: The prevention effort has a non-increasing marginal impact on the loss distribution, i.e.
the map $e \mapsto \overline{q}_{X_e}(u)$ is convex, for any $u \in (0,1)$.

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- $\alpha^\star$ can only take the values 0 and 1.
- There exists $\theta_M > 0$ such that for $\theta \geq \theta_M$, the buyer stops purchasing insurance.
- The two main conclusions of Ehrlich and Becker (1972) do not necessarily hold true in this framework. They depend on the relative impact of the effort on the risk and on the prices (For instance $e \mapsto \frac{\rho_A(X_e)}{\mathbb{E}[X_e]}$ increasing).

Perspectives

- Time dynamic version of this problem (stochastic control).
- Non-proportional insurance contracts (layers).
- Losses given by a vector of dependent risks (copulas).
- Initial wealth effects: Expected utility criteria and/or Cash sub-additive risk measures (El Karoui, Ravanelli 2009)

Thank you for your attention

Figure: Happy (3×5^2)-th anniversarry!