Prevention efforts and insurance demand under coherent risk measures

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Conférence en l’honneur des 3 × 25 ans de Nicole El Karoui
Study of actions taken by agents to **reduce their risk exposure**.

1. Transfer the risk to a counterparty (Well studied by Nicole: risk transfer, weather derivatives, inf-convolution of risk measures...)
   - Insurance
   - Reinsurance
   - Insurance linked securities (Cat bonds...)

**Motivations**
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Study of actions taken by agents to reduce their risk exposure.

1. Transfer the risk to a counterparty (Well studied by Nicole: risk transfer, weather derivatives, inf-convolution of risk measures...)
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2. Take prevention actions.
   - Optimal choice between market insurance and prevention activities.
   - Self-protection: reduction of the probability to suffer a claim.
   - Self-insurance: reduction of the claim amount.
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- market insurance and self-protection could be *complements*,

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- Ehrlich and Becker (1972): **Expected utility** framework. They showed that
- market insurance and self-insurance are **substitutes**, 
- market insurance and self-protection could be **complements**, depending on the level of the loss probability.
- Led to many discussions and extensions on the optimal individual behaviour with respect to prevention (Bleichrodt, Briys, Chiu, Courbage, Dionne, Eeckhoudt, Gollier, Konrad, Rey, Schlesinger, Skaperdas, Treich etc.)
A model inspired by contract theory

**Principal-agent** type model where:

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  - $\alpha$ = proportion of losses paid by the insurer and
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  - $\pi(X) := (1 + \theta)\mathbb{E}[X]$, with $\theta \geq 0$.
- Losses = r.v. $(X_e)_{e \in (0, +\infty)}$ whose distributions form a family of probability measures which is decreasing for the first order stochastic dominance.
The buyer’s problem

**Goal:** minimize his risk.

\[
\inf \left( \alpha, e \right) \in [0, 1] \times (0, \infty) \{ (1 - \alpha) \rho_A(Xe) + \alpha (1 + \theta) E_P[Xe] + c(e) \}.
\]

where \( \rho_A \) is a given law invariant coherent risk measure, and \( c \) is a non-decreasing convex function.
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The seller’s problem

Assume that \((\alpha^*(\theta), e^*(\theta))\) solves the buyer’s problem.

**Goal:** offers linear contracts and safety loading prices by solving

\[
\inf_{\theta \in A} \{ \alpha^*(\theta) \rho P(X) - \alpha^*(\theta)(1 + \theta) E[P[X e^*(\theta)]] \}
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where \(\rho_P\) is a given law invariant coherent risk measure, and \(\mathcal{A}\) is the set of prices accepted by the buyer.
Main Assumption: The prevention effort has a non-increasing marginal impact on the loss distribution, i.e.

the map \( e \mapsto \bar{q}_{X_e}(u) \) is convex, for any \( u \in (0, 1) \).
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- There exists $\theta_M > 0$ such that for $\theta \geq \theta_M$, the buyer stops purchasing insurance.
- The two main conclusions of Ehrlich and Becker (1972) do not necessarily hold true in this framework. They depend on the relative impact of the effort on the risk and on the prices (For instance $e \mapsto \frac{\rho_A(X_e)}{E[X_e]}$ increasing).
• Time dynamic version of this problem (stochastic control).
• Non-proportional insurance contracts (layers).
• Losses given by a vector of dependent risks (copulas).
• Initial wealth effects: Expected utility criteria and/or Cash sub-additive risk measures (El Karoui, Ravanelli 2009)
Thank you for your attention

**Figure:** Happy \((3 \times 5^2)\)-th anniversary!