Insurance capacity

Jean Charles Rochet[1]
joint work with:Elisa Luciano [2]

[1]GFRI, University of Geneva and SFI

[2]University of Torino, Collegio Carlo Alberto - Italy

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Introduction

Motivation

- Fact 1: Insurance capacity, namely how much insurance is a company (or the whole industry) ready to supply, is important in practice but under-researched by academics.
- Fact 2: empirically, insurance companies' behave as risk-averse agents, with risk aversion decreasing in capitalization/book equity. Relationship is causal (Ge and Weisbach, 2019).

Objective and preview of main results

- We build a simple structural model of insurance capacity with financial frictions. Both demand and supply are endogenized.
- We find that, even if shareholders are diversified, insurance companies behave as risk-averse agents. Risk aversion explodes when capital constraints hit, and decreases when equity increases.
- Results are robust to market concentration (monopoly to perfect competition).

Core model

Households

Households live from t to $t + dt$, receive a random labor income

$$di_t = adt + \sigma dZ_t$$

where $a, \sigma \in \mathbb{R}^{++}$ and $Z_t, t \geq 0$, is a *one*-dimensional Wiener process, and have mean variance preferences with risk aversion α. Demand for insurance:

$$y_t = 1 - \frac{\pi_t}{\alpha\sigma^2} \tag{1}$$

or, equivalently, the premium as a function of the demand

$$\pi_t = \alpha\sigma^2 (1 - y_t) \tag{2}$$

Insurance company: monopoly case

Objective function: expected sum of discounted dividends

$$\mathbb{E} \int_0^{+\infty} \exp(-\lambda t) dC_t$$

where the state variable W_t (reserves, also book value of equity) evolves as:

$$dW_t = rW_t dt - dC_t + y_t(\pi_t dt + \sigma dZ_t) \tag{3}$$

Insurer sets capacity y and dividend policy dC so as to solve:

$$\mathcal{P} \begin{cases} J(W_0) \triangleq \max_{C \geq 0, y} \mathbb{E} \int_0^{+\infty} \exp(-\lambda t) dC_t \\ \text{subject to (3)} \\ W_t \geq 0 \end{cases}$$

Insurance company, cont'ed

J solves the Bellman equation

$$\lambda J = rWJ' + \max_{C \geq 0} dC\,(1 - J') + \sigma^2 \max_y \left\{ J'\alpha y(1-y) + \frac{1}{2} J'' y^2 \right\}$$

under the BCs

$$J(0) = 0 \quad (4)$$
$$J'(W^*) = 1 \quad (5)$$
$$J''(W^*) = 0 \quad (6)$$

where W^* is the level of wealth above which dividends are distributed (reflecting boundary). The FOC for y gives

$$y(W) = \frac{\alpha}{2\alpha - J''/J'} > 0 \quad (7)$$

Substituting into the Bellman equation we get

$$\lambda J = rWJ' + \frac{\alpha^2 \sigma^2 J'^2}{2\left[2\alpha J' - J''\right]} \quad (8)$$

Insurance companies, cont'ed

The key to solving the ODE for J consists in transforming it into

$$y'(W) = \alpha(1 - 2y(W))(1 + \frac{2rW}{\sigma^2 y(W)\alpha} + \frac{2(\lambda - r)}{\alpha\sigma^2} \tag{9}$$

under the BCs

$$y(0) = 0, y(W^*) = 1/2. \tag{10}$$

Equilibrium

Theorem 1

This boundary value problem has a unique solution.

Theorem 2

The function $J(\cdot) = \int_0^W J'(s)ds$ is the value function of the monopoly problem $\mathcal{P}$ and $y(\cdot)$ in Theorem 1 is the optimal solution to this problem. The premium at time t is

$$\pi_t = \alpha\sigma^2 (1 - y_t)$$

Insurance capacity and premium

The following pictures show the typical behavior of y and π, for parameter values:

$$\alpha = 2, \lambda = 4\%, r = 3\%, \sigma = .2.$$

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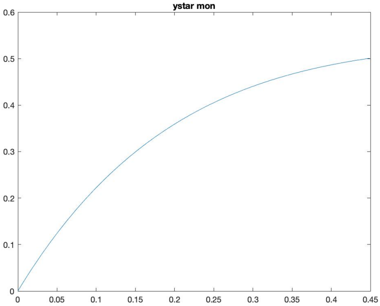
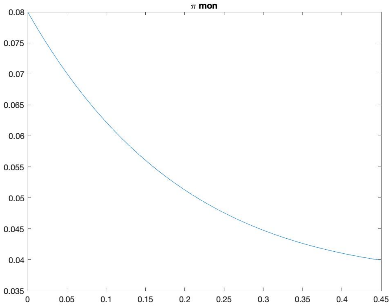


(a) Insurance capacity



(b) Insurance premium

Ergodic behavior of the equilibrium

Stationary distribution of insurer equity and capacity

Proposition 3

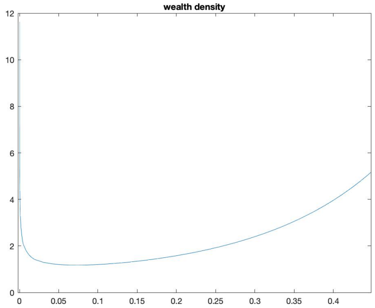
Equity of insurer admits a stationary distribution with density proportional to

$$\frac{1}{(y(W))^2\,\sigma^2} \times exp\left(-\frac{2r}{\sigma^2}\int_W^{W^*}\frac{u}{(y(u))^2}du - 2\alpha\int_W^{W^*}\frac{1-y(u)}{y(u)}du\right) \qquad (11)$$

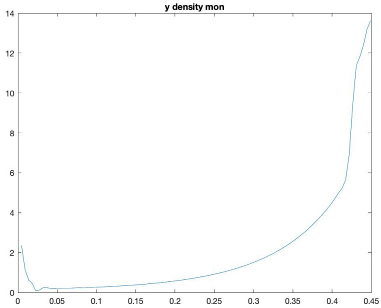
.

Similarly for capacity, premiums and prices.

Ergodic densities of equity and capacity



(a) Wealth density



(b) Capacity density

Competition

Define

$$J(W,\hat{W}) \triangleq \text{Max}_{\hat{C}_t \geq 0, \hat{y}_t} \mathbb{E} \int_0^{+\infty} \exp(-\lambda t) d\hat{C}_t$$

The second-order PDE for J turns into the first-order ODE for y_c :

$$y_c'(W) = \frac{2rW}{\sigma^2}\frac{1-y_c(W)}{y_c(W)} + \alpha\left(1-y_c(W)\right)^2 + 2\frac{\lambda - r}{\alpha\sigma^2} \tag{12}$$

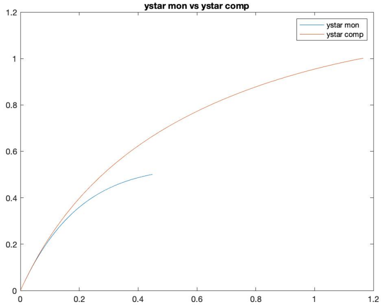
under $y_c(0) = 0$ and $y_c(W^*) = 1$. Fully analytical sol for $r = 0$.

Proposition 4

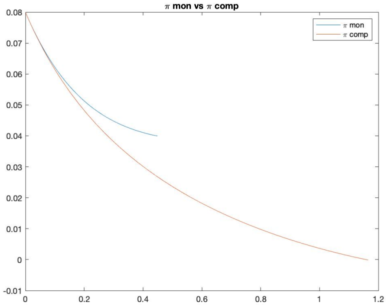
y_c greater than under monopoly, and premiums are lower.

Competition versus monopoly

The pictures below present the behavior of the competitive equilibrium, with the same parameter values adopted for the monopolistic case. Here $W^* = 1.1651$



(a) Insurance capacity, monopoly vs competition

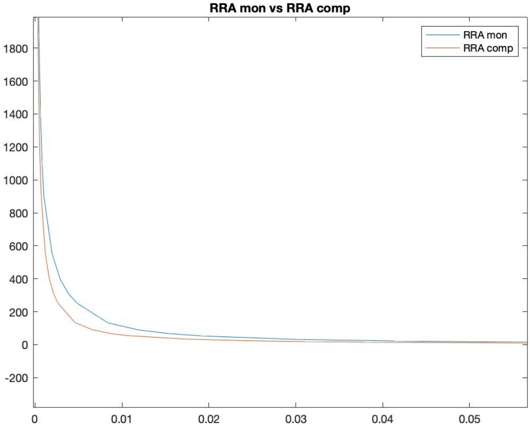


(b) Insurance premium, monopoly vs competition

Relative risk aversion *RRA*

Insurers are risk averse, and RRA is decreasing with capacity

$$RRA = -\frac{q'}{q} = \alpha\left(\frac{1}{y} - 1\right)$$



Competition versus monopoly, cont'ed

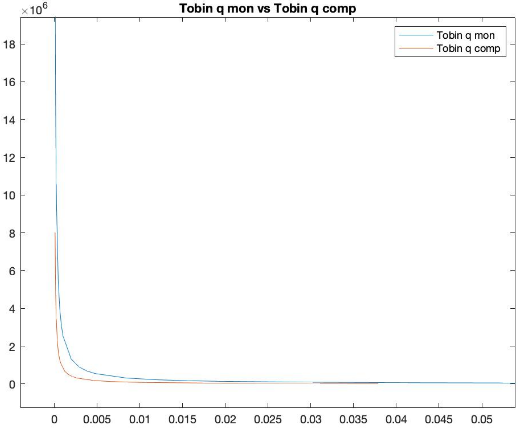


Figure. Tobin's q

Recapitalization

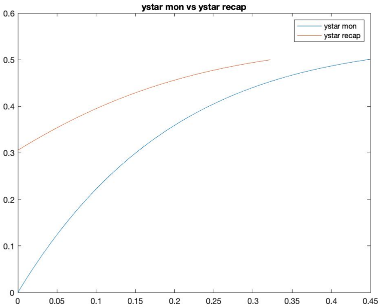
Assume that, when insurers equity becomes very small, they can issue new equity at marginal cost c. The boundary condition for the insurer's Bellman equation at $W = 0$ becomes

$$J'(0) = 1 + c$$

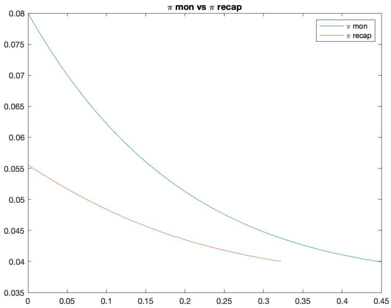
Proposition 5

y greater than without recapitalization, premiums are lower and dividends are distributed more often (W^* is smaller).

Recapitalization equilibrium



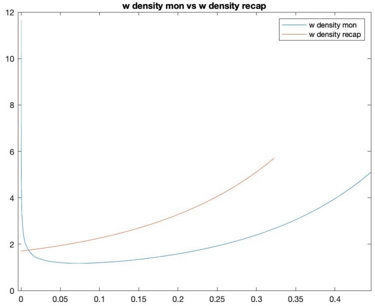
(a) Insurance capacity



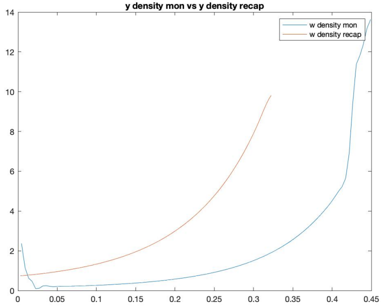
(b) Insurance premium

Recapitalization equilibrium, cont'ed

There is an "underwriting cycle"



(a) Equity density



(b) Capacity density

Long-lived assets properties with recap

Economic implications

- Insurance capacity is an increasing and concave function of insurers' capitalization.
- Insurers's risk aversion is a decreasing function of capitalization.
- Competition boosts capacity.
- When costly recapitalization is possible, capacity is higher but there are underwriting cycles.

http://sites.carloalberto.org/luciano/
http://www.carloalberto.org/lti/

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Solving for Prices

The solution to the ODE (**??**) obtains by the change of variable

$$F(W) \triangleq \frac{\sigma^2 (y(W))^2}{2} S''(W)$$

$$F'(W) = -\alpha^2 \sigma^2 (1 - 2y(W)) \left(1 + \frac{2rW}{\sigma^2 y(W) \alpha}\right) - 2(\lambda - r) - \frac{2rW}{\sigma^2 (y(W))^2} F(W) \tag{13}$$

with

$$F(0) = \frac{\sigma^2 (y(0))^2}{2} S''(0) = 0 \tag{14}$$

which translates into $F(\varepsilon) = B\varepsilon$, where

$$B = \frac{\sigma^2 b \left(-\alpha^2 \sigma^2 b - 2r\alpha - 2b(\lambda - r)\right)}{\sigma^2 b^2 + 2r} < 0$$

Solving for prices, cont'ed

Having solved for F, the price level S can be reconstructed integrating twice

$$S''(W) = \frac{2}{\sigma^2 \left(y(W)\right)^2} F(W)$$

under the BCs $S(0) = \frac{\bar{D} - \pi^0}{r}$ and $S'(W^*) = 0$

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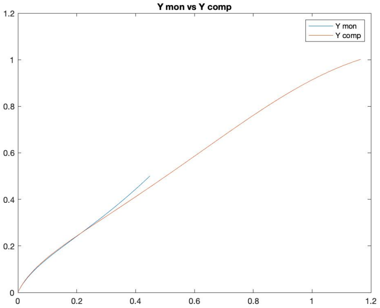
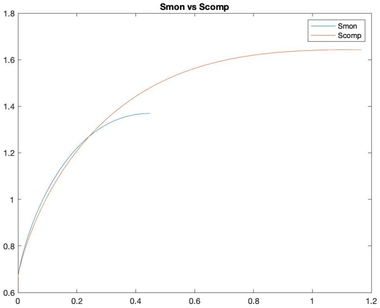
Competition versus monopoly, cont'ed

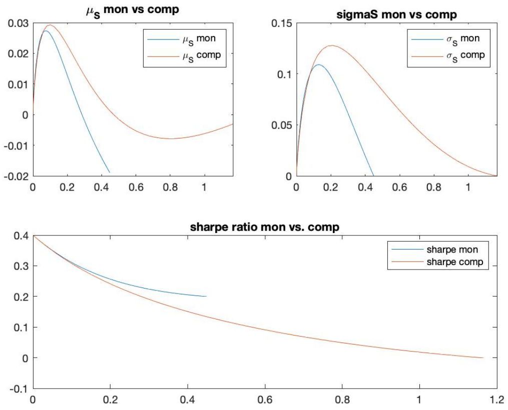


(a) Long-lived insurance capacity



(b) Long-lived insurance prices

Competition versus monopoly, cont'ed



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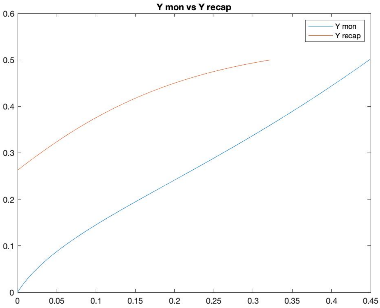
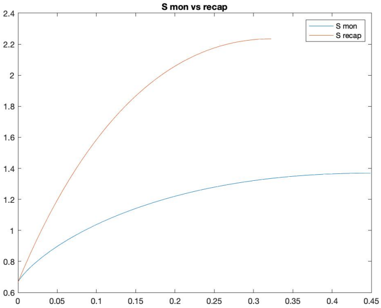
Recapitalization equilibrium, cont'ed



(a) Long-lived insurance capacity



(b) Long-lived insurance prices

Recapitalization equilibrium, cont'd

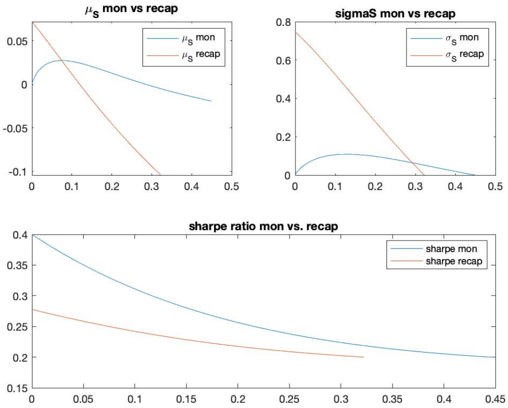


Figure. Drift, diffusion, market price of risk

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