Size Discovery Trading and the Cost of Market Fragmentation

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Based on work with Samuel Antill and Haoxiang Zhu

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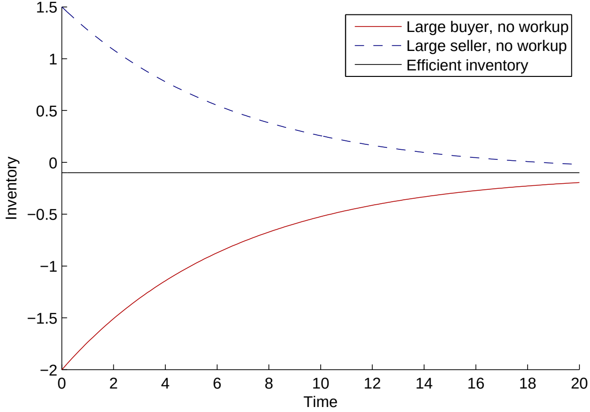
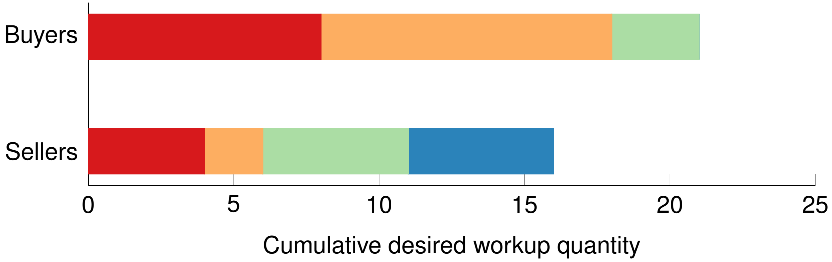
Avoidance of price impact delays efficient trades



Vayanos (1999), Rostek and Weretka (2015), Du and Zhu (2017), Duffie and Zhu (2017).

A workup session involving 3 buyers and 4 sellers.



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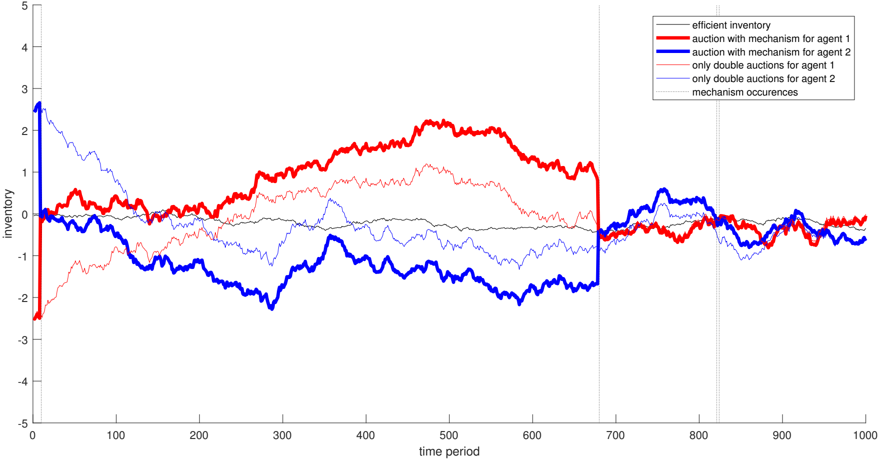
- Scope for regulation. In 2018, European MiFiD II regulations capped dark-pool equities trade volume at 8%, and 4% for each platform.

Main findings

1. Size discovery is highly effective for avoiding price-impact costs and can dramatically improve allocations whenever it is run.
2. The prospect of size discovery, however, reduces exchange market trade volumes and depth.
3. The net effect, as modeled, is a reduction in overall allocative efficiency.
4. Ex ante, every investor would strictly prefer the absence of size discovery.

Model Timeline

Inventory paths with and without size-discovery sessions



Static size-discovery primitives

- $n \geq 3$ traders with initial excess inventories $z_0^1, \ldots, z_0^n$, with $z_0^i \in \mathcal{F}_i$.
- Average excess inventory $\bar{Z} = (z_0^1 + \cdots + z_0^n)/n$.
- Continuation value of excess inventory:

$$V^i(z^i, \bar{Z}) = u^i(\bar{Z}) + \beta(\bar{Z})z^i - K\left(z^i - \bar{Z}\right)^2,$$

  where u^i, β are functions and $K > 0$ is a scalar.
- The unique efficient allocation: $z^i = \bar{Z}$.

Static size-discovery game

- Suppose (for now) that $\bar{Z}$ is publicly observable.
- Trader i submits an inventory report $\hat{z}^i$.
- Given reports $\hat{z} = (\hat{z}^1, \ldots, \hat{z}^n)$, trader i gets some cash transfer $T^i(\hat{z}, \bar{Z})$ and some asset transfer $Y^i(\hat{z})$.
- Taking $\hat{z}^{-i}$ as given, trader i solves

$$\sup_{\tilde{z}} \mathbb{E}\left[V^i(z_0^i + Y^i((\tilde{z}, \hat{z}^{-i})), \bar{Z}) + T^i((\tilde{z}, \hat{z}^{-i}), \bar{Z}) \,|\, \mathcal{F}^i \right].$$

An efficient mechanism design for size discovery

- Asset transfer

$$Y^i(\hat{z}) = \frac{\sum_{j=1}^{n} \hat{z}^j}{n} - \hat{z}^i.$$

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- Cash transfer

$$T^i(\hat{z}, \bar{Z}) = \underbrace{\kappa_1(\bar{Z})}_{\text{frozen price}} \hat{z}^i + \kappa_0 \left(n\,\kappa_2(\bar{Z}) + \sum_{j=1}^{n} \hat{z}^j \right)^2 + \kappa_1(\bar{Z})\kappa_2(\bar{Z}) + \frac{\kappa_1^2(\bar{Z})}{4\kappa_0 n^2},$$

where $\kappa_1(\,\cdot\,)$ and $\kappa_2(\,\cdot\,)$ are affine and $\kappa_0 < 0$ is a constant.

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- Budget feasibility: $\sum_i T^i(\hat{z}, \bar{Z}) \leq 0$ for any $\bar{Z}$ and $\hat{z} \in \mathbb{R}^n$.

Static size-discovery mechanism-design results

Result (Antill and Duffie, 2019): For unique $\kappa_0, \kappa_1, \kappa_2$, this size-discovery mechanism is:

- Strategy proof: Reporting $\hat{z}^i = z_0^i$ is a strictly dominant strategy.
- Ex-post IR: For any $z_0 \in \mathbb{R}^n$ and for the equilibrium strategies $\hat{z}^i = z_0^i$,

$$V^i(z_0^i, \bar{Z}) \leq V^i(z_0^i + Y^i(z_0), \bar{Z}) + T^i(z_0, \bar{Z}).$$

- Efficient: $z_0^i + Y^i(\hat{z}) = \bar{Z}$.

Remaining primitives of the dynamic model

- The cumulative inventory shock of trader i is a zero-mean Lévy process H^i.
- At time $\mathcal{T} \sim exp(r)$, the asset pays an independent amount with mean v.
- Almgren-Chriss holding cost for excess-inventory process z is $\gamma \int_0^{\mathcal{T}} z_t^2 \, dt$.
- Without trade, the total value to trader i is therefore

$$E\left[v z_{\mathcal{T}}^i - \gamma \int_0^{\mathcal{T}} (z_t^i)^2 \, dt \right],$$

where $z_t^i = z_0^i + H_t^i$.

The exchange: A dynamic double-auction market

- At a given price p, in state ω at time t, trader i demands the asset at some chosen quantity rate $\mathcal{D}_t^i(\omega, p) \in \mathbb{R}$.

- For a given price process ϕ, the total payment by trader i in state ω is thus

$$\int_0^T \phi_t(\omega)\mathcal{D}_t^i(\omega, \phi_t(\omega))\, dt.$$

- The associated excess inventory process of trader i is

$$z_t^i = z_0^i + \int_0^T \mathcal{D}_t^i(\phi_t)\, dt + H_t^i.$$

Strategic avoidance of price impact

- Vayanos (1999), Rostek and Weretka (2015), Du and Zhu (2017).
- At price p, trader i assumes that each trader $j \neq i$ demands

$$\mathcal{D}_t^j(\omega, p) = a + bp + cz_t^j(\omega),$$

  for some given a, $b < 0$, and c.
- Continuous-time version of the Du-Zhu equilibrium for the demand-function submission game, with price process ϕ.

Augmenting with size-discovery sessions

- Size-discovery sessions are held at Poisson arrivals with mean frequency λ.
- In equilibrium, the average excess inventory $\bar{Z}_t$ can be inferred from the exchange price $\phi_t = \Phi(\bar{Z}_t)$.
- A mechanism session at time t generates the cash transfer

$$\hat{T}^i(\hat{z}_t, \phi_t) = \phi_t\, \hat{z}^i + \kappa_0 \left(-n\delta(\phi_t) + \sum_{j=1}^{n} \hat{z}^j \right)^2 - \phi_t \delta(\phi_t) + \frac{\phi_t^2}{4\kappa_0 n^2}.$$

Equilibrium with exchange trading and mechanism sessions

- Trader i submits exchange demand process $\mathcal{D}$ and a mechanism report process $\hat{z}^i$, generating the excess inventory process

$$z_t^i = z_0^i + \underbrace{\int_0^t \mathcal{D}_s \, ds}_{\text{Exchange trade}} + \underbrace{\int_0^t Y^i(\hat{z}_s) \, dN_s}_{\text{Mechanism trade}} + H_t^i.$$

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- The stochastic control problem of trader i, given other traders' strategies, is

$$\sup_{\mathcal{D}, \hat{z}^i} E\left[z_T^i v - \int_0^T \phi_t^{\mathcal{D}} \mathcal{D}_t \, dt + \int_0^T T^i(\hat{z}_t, \phi_t^{\mathcal{D}}) \, dN_t - \gamma \int_i^T (z_t^i)^2 \, dt \right].$$

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- Equilibrium: market clearing, consistent conjectures, and agent optimality (incentive compatibility and mechanism IR).

Augmenting with mechanism sessions reduces allocative efficiency

Proposition (Antill and Duffie)

1. Above a stated mean frequency $\bar{\lambda}$ of size-discovery sessions, exchange trading breaks down.
2. For any $\lambda < \bar{\lambda}$, there are 2 linear equilibria. In the more efficient equilibrium, welfare (indeed, *every* trader's value) is strictly decreasing in λ.

Policy-related observations

- With competing platform operators, entry of a size-discovery platform is profitable but (in our model) socially harmful.
- As size-discovery sessions become more frequent, exchange volume and depth decline.
- If size discovery is available, traders will use it even though they are better off (in our model) without it.
- Scope for regulation. MiFiD II caps dark pool trading volume.
- The policy-relevant empirical evidence is limited to equities, and mixed, Buti, Rindi, and Werner (2011), DeGryse, De Jong, and Kervel (2015), Nimalendran and Ray (2014), Farley, Kelley, and Puckett (2017).