LoLitA

Dynamic models for human
Longevity with Lifestyle Adjustments

On the longevity adventure with Nicole and LoLitA

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Travaux conjoints avec N. El Karoui, Y. Salhi & LoLitA (entre autres)

GDPR Statement

Due to GDPR there will be no picture of Nicole during this talk

Modified GDPR Statement

Due to GDPR there will be only one picture of Nicole during this talk

Why a talk about football at the WISE day?

Age pyramids, England

- Excerpt from Sarah Kaakai PhD thesis

(Thèse de doctorat de l’UPMC encadrée par Nicole El Karoui, 2017)

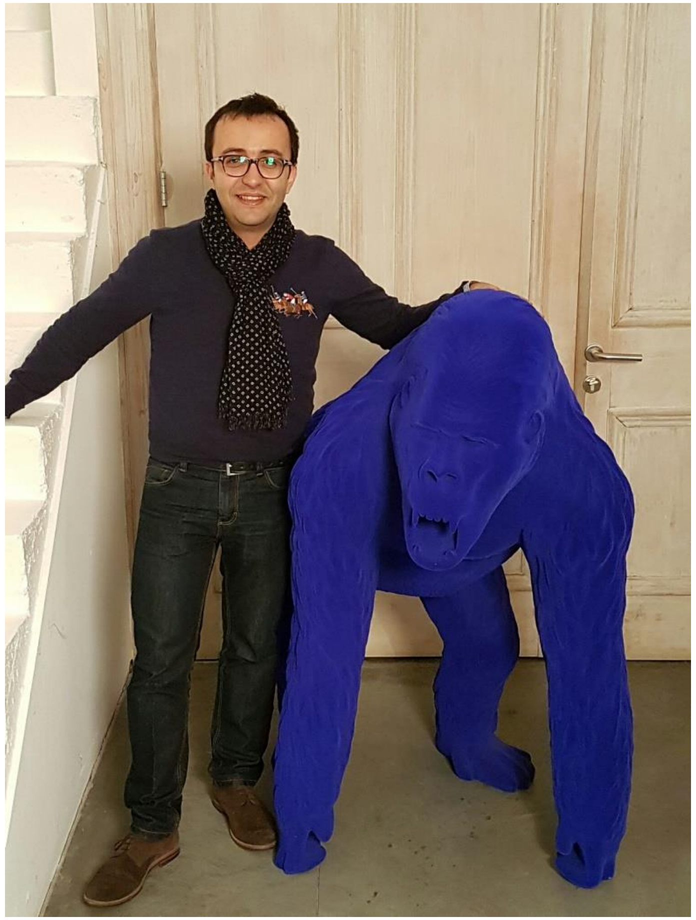
(a) England population, 2001 (b) England population, 2015

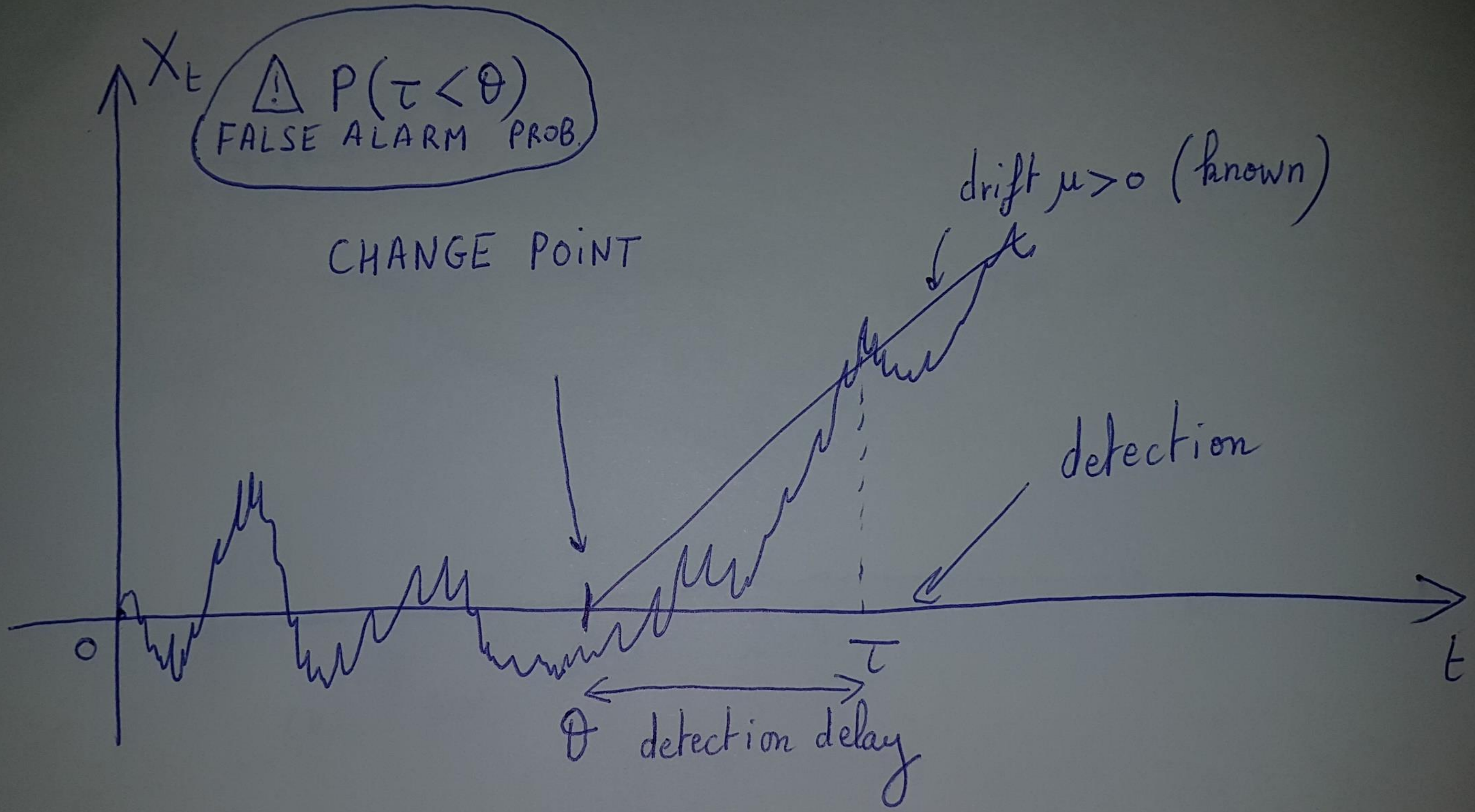
(c) Least deprived quintile ($$$$$), 2001 (d) Least deprived quintile ($$$$$), 2015

(e) Most deprived quintile ($), 2001 (f) Most deprived quintile ($), 2015

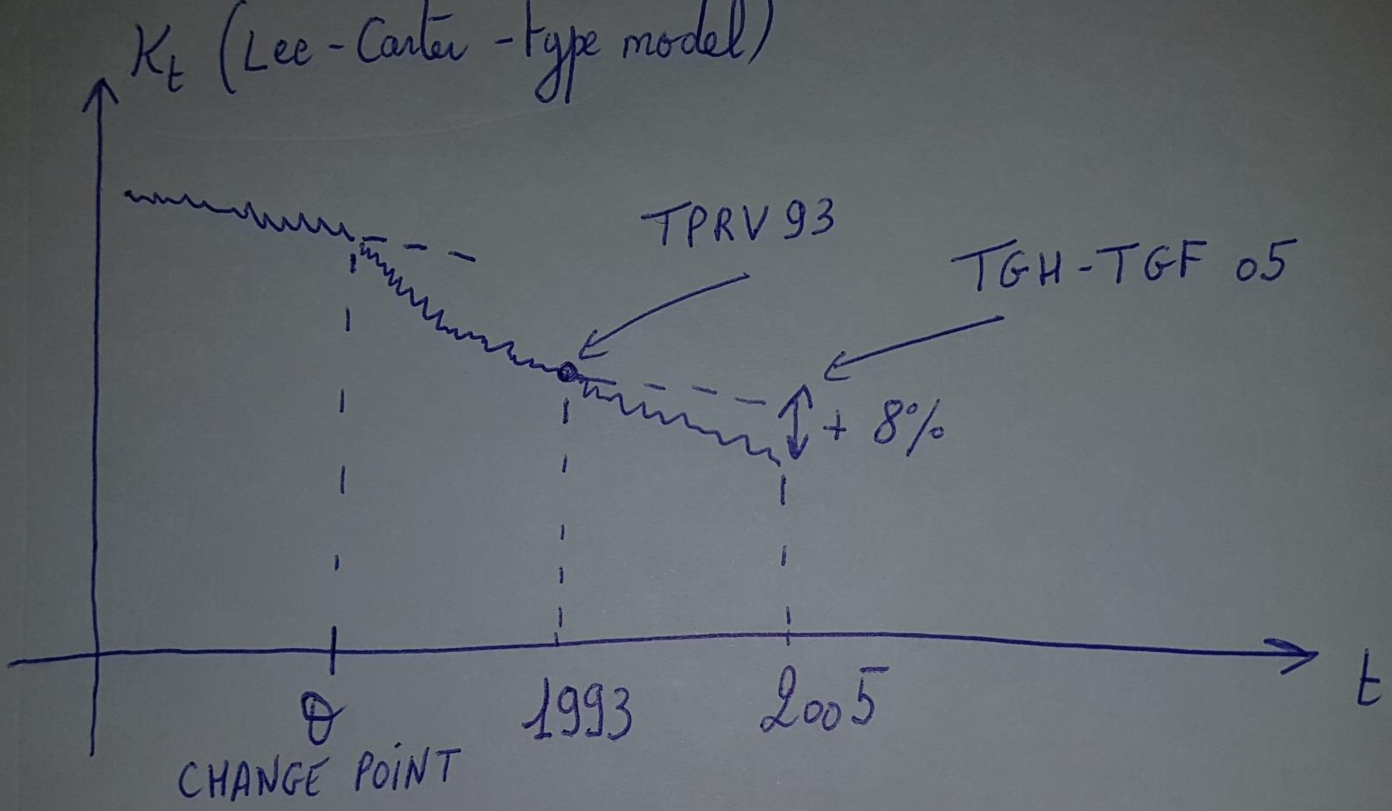
Fig. 1.3 Age pyramids in 2001 and 2015 (Figure 4.1)

Yahia Salhi





X_t
⚠ $P(\tau < \theta)$
FALSE ALARM PROB.
drift $\mu > 0$ (known)
CHANGE POINT
detection
0
τ
t
θ
detection delay



K_t (Lee-Carter-type model)
TPRV 93
TGH-TGF 05
+ 8%
t
θ
CHANGE POINT
1993
2005

Bayesian setup for random change-point

Brownian framework with abrupt change in the drift

- Based on the conditional distribution of the time of change,
- Formulated as an optimal stopping problem
- Page(1954), Shiryaev(1963), Roberts(1966), Beibel(1988), Moustakides (2004), and many others...

Poisson framework with abrupt change in intensity

- Based on the conditional distribution of the time of change, with exponential or geometric prior distribution
- More recent studies : Gal (1971), Gapeev (2005), Bayraktar (2005, 2006), Dayanik (2006) for compound Poisson, Peskir, Shyriaev(2009) and others

Mathematical settings

We consider a portfolio of insured population:

- Let $N = (N_t)_{t \geq 0}$ be a counting process indicating the deaths of policyholders and $\lambda = (\lambda_t)_{t \geq 0}$ its intensity.
- The counting process N_t, is available sequentially through the filtration $\mathcal{F}_t = \sigma\{N_s, 0 < s \leq t\}$.
- We suppose that the insurance company relies on a Cox-like model to project her own experienced mortality:

$$\lambda_t = \underline{\rho}\lambda_t^0,$$

  - λ_t^0 is a reference intensity and $\underline{\rho}$ is a positive parameter.
  - λ^0 is considered deterministic and may refer whether to a projection of national population/best estimate...

Model risk/parameter uncertainty: **Change-point**

$$\lambda_t = \mathbf{1}_{\{t < \boldsymbol{\theta}\}}\underline{\rho}\lambda_t^0 + \mathbf{1}_{\{t \geq \boldsymbol{\theta}\}}\overline{\rho}\lambda_t^0.$$

Without loss of generality we can assume that $\underline{\rho} = 1$ and let $\boldsymbol{\rho} = \overline{\rho} > 1$.

Probabilistic formulation

Let $\mathbb{P}_\theta$ (resp. $\mathbb{E}_\theta[\cdot]$) be the probability measure (resp. expectation) induced when the change takes place at time θ

Example

- For $\theta = 0$, the process is *out-of-control*
- For $\theta = \infty$, the process is *in-control*

Detect the change-point θ as quick as possible while avoiding false alarms

Optimality Criteria, Lorden (1971)-like

- The detection delay $\mathbb{E}_\theta\left[(N_\tau - N_\theta)^+ \middle| \mathcal{F}_\theta\right]$
- The frequency of false alarm $\mathbb{E}_{\infty}[N_\tau]$

Optimization Problem

Optimization Problem

$$\text{Find } \tau^* \text{ such that } C(\tau^*) = \inf_\tau \textbf{sup}_{\theta \in [0,\infty]} \textbf{ess sup} \mathbb{E}_\theta \left[(N_\tau - N_\theta)^+ \middle| \mathcal{F}_\theta \right]$$
$$\text{subject to } \mathbb{E}_\infty[N_\tau] = \omega.$$

Assumption

1. $\int_0^t \lambda_s ds < \infty, \quad \mathbb{P}_\infty, \mathbb{P}_0$-a.s.
2. $N_\infty = \infty \qquad \mathbb{P}_\infty, \mathbb{P}_0$-a.s.

Optimality of the Cusum procedure (1/7)

Let the Radon-Nikodym density of $\mathbb{P}_0$ with respect to $\mathbb{P}_\infty$ be defined as

$$\left.\frac{d\mathbb{P}_0}{d\mathbb{P}_\infty}\right|_{\mathcal{F}_t} = \exp U_t,$$

where $U_t = \log(\rho)N_t + (1-\rho)\int_0^t \lambda_s^0 ds$ is the log-likelihood ratio.
Let $V(x)$ be the CUSUM process; with head-start $0 \leq x < m$; defined as

$$V_t(x) = U_t - (-x) \wedge \underline{U}_t \tag{1}$$

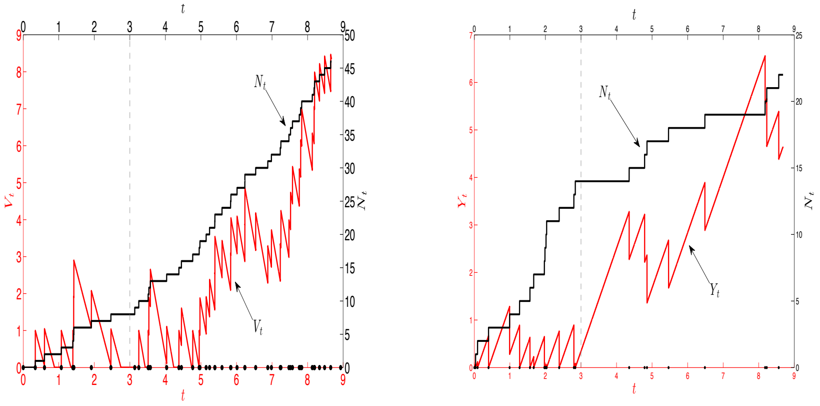
where $\underline{U}_t$ is the running infimum of U, i.e. $\underline{U}_t = \inf_{s\leq t} U_s$.
The process $V(x)$ measures the size of the drawup, comparing the present value of the process U to its historical infimum $\underline{U}$.
Let $\tau_m(x)$ be the fist hitting time of $V(x)$ of the barrier m, i.e.

$$\tau_m(x) = \inf\{t \geq 0, V_t(x) \geq m\}.$$

Theorem

If $\mathbb{E}_\infty[N_{\tau_m(0)}] = \omega$ *then* $\tau_m(0)$ *is optimal, i.e.* $\inf_\tau C(\tau) = C(\tau_m(0))$

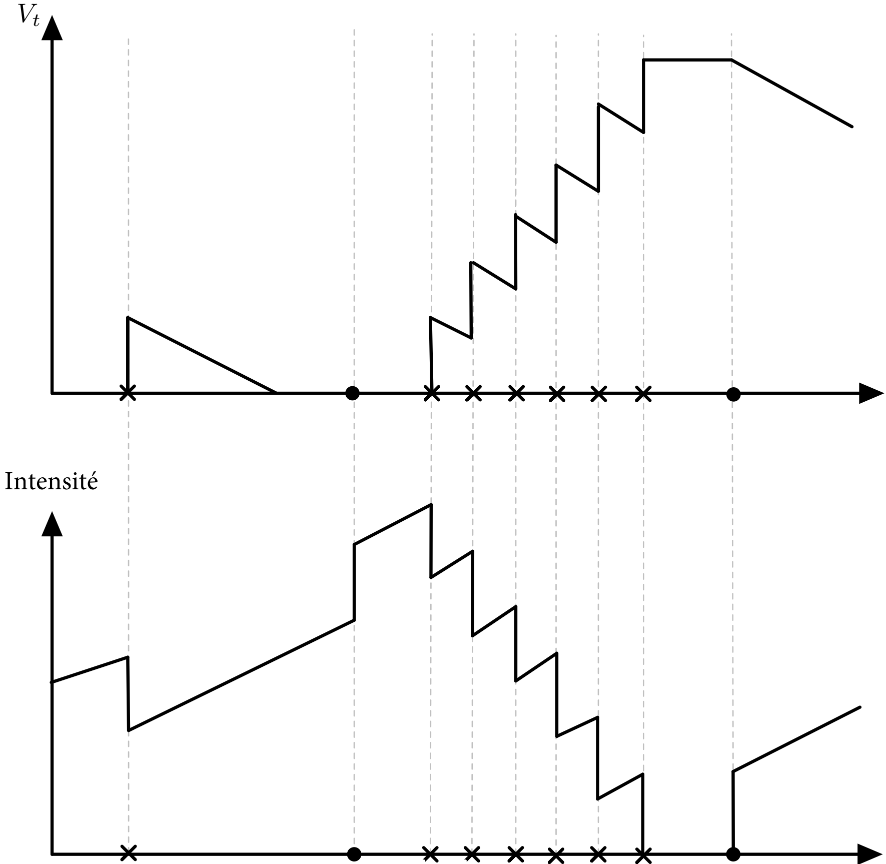
Typical paths with change of regime at date 3



(a) Processes N and V_t

(b) Processes N and Y_t

Figure: Sample paths, for $\rho = 1.5$, of the cusum processes N, V^ρ (left) and N, Y_t^ρ for $\rho = 0.5$ (right).


V_t
Intensité

Monitoring Mortality

Sounding an alarm for the change $\rho^{\text{Hyp}} \to \rho^{\text{Target}}$

- We simulate deaths on the portfolio with different levels $\rho^{\text{Target}} = 95\%, 90\%$ and 85% s.t.

$$D(x, t) \sim \text{Pois}(\rho^{\text{Target}} \times L(x, t) \times \mu^{\text{ERM00}}(x, t))$$

- We suppose that *the actuary* made an assumption of $\rho^{\text{Hyp}} = 100\%$
- We set-up the monitoring/surveillance on the observed deaths and try to detect a change from $\rho^{\text{Hyp}} = 100\%$ to $\rho^{\text{Target}} = 95\%, 90\%$ and 85% respectively.
- We test different sizes of the portfolio small sized 1000, 5000 and a (relatively) large 10000 and compare the results

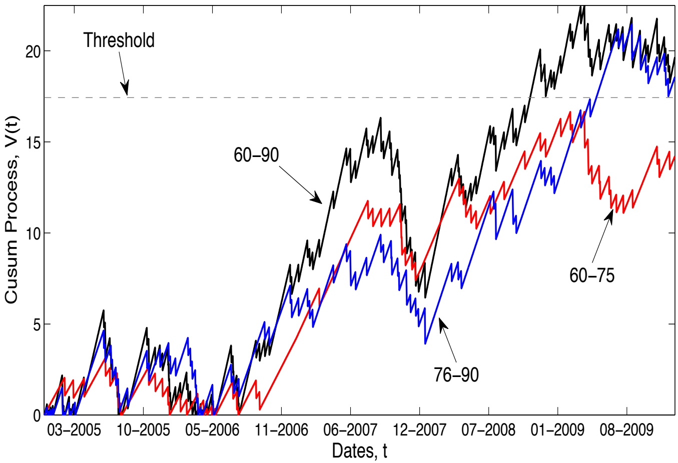
How to choose parameter Rhô?

	females				males			
	Doubled improvements		Mortality level at 80% of the expected		Doubled improvements		Mortality level at 80% of the expected	
	pension value	interest rate	pension value	interest rate	pension value	interest rate	pension value	interest rate
55	+5.4%	+32bp	+3.1%	+19bp	+6.7%	+42bp	+3.7%	+24bp
65	+5.76%	+43bp	+4.7%	+36bp	+7%	+57bp	+5.7%	+48bp
75	+5.2%	+55bp	+7.6%	+80bp	+6.3%	+74bp	+9.1%	+107bp
85	+3.6%	+60bp	+13.2%	+207bp	+4.3%	+84bp	+15.4%	+281bp

TABLE: TGH05/TGF05 with flat interest rate of 3%

Monitoring Mortality

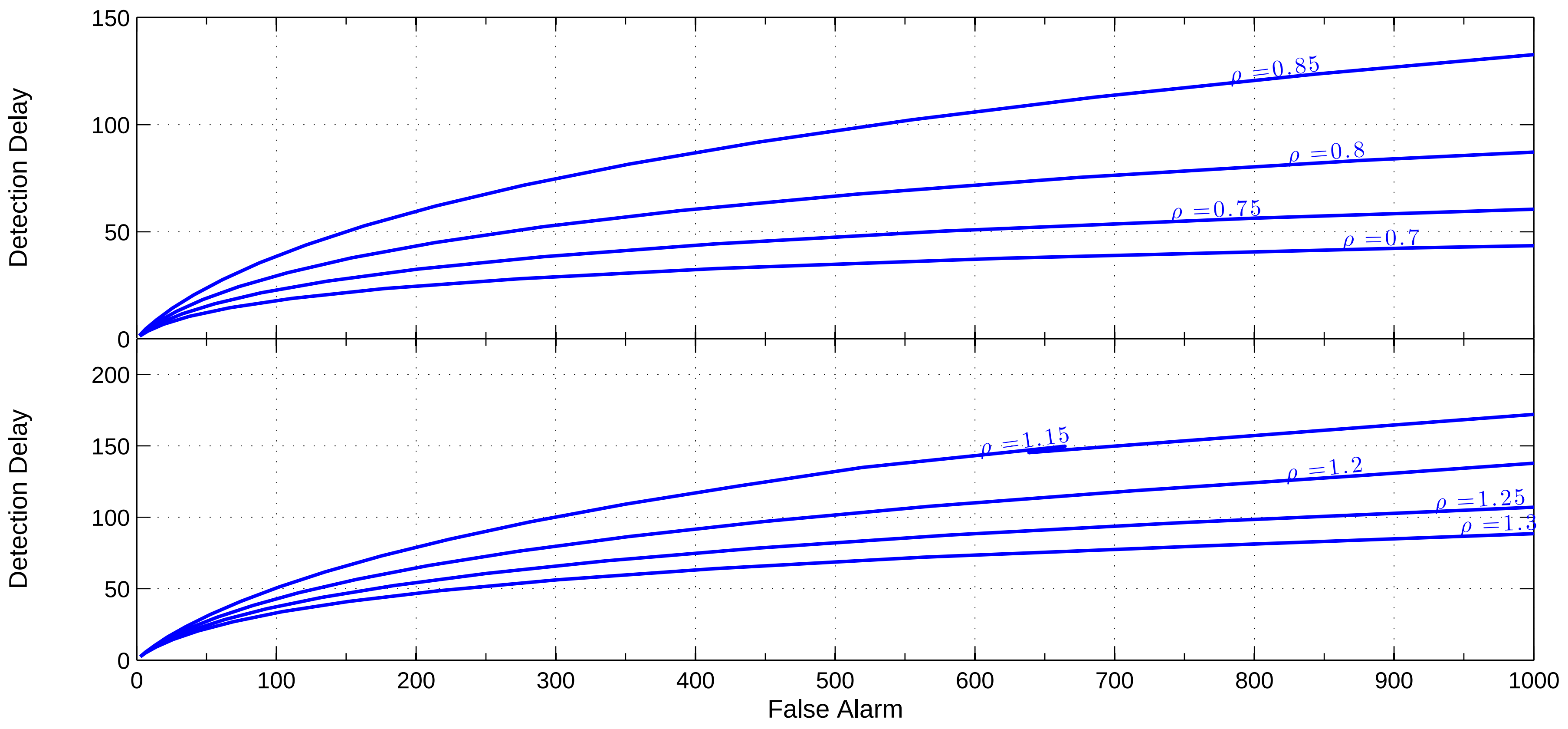
Sounding an alarm for the change $\rho^{\text{Hyp}} = 100\% \rightarrow \rho^{\text{Targer}} = 95\%$



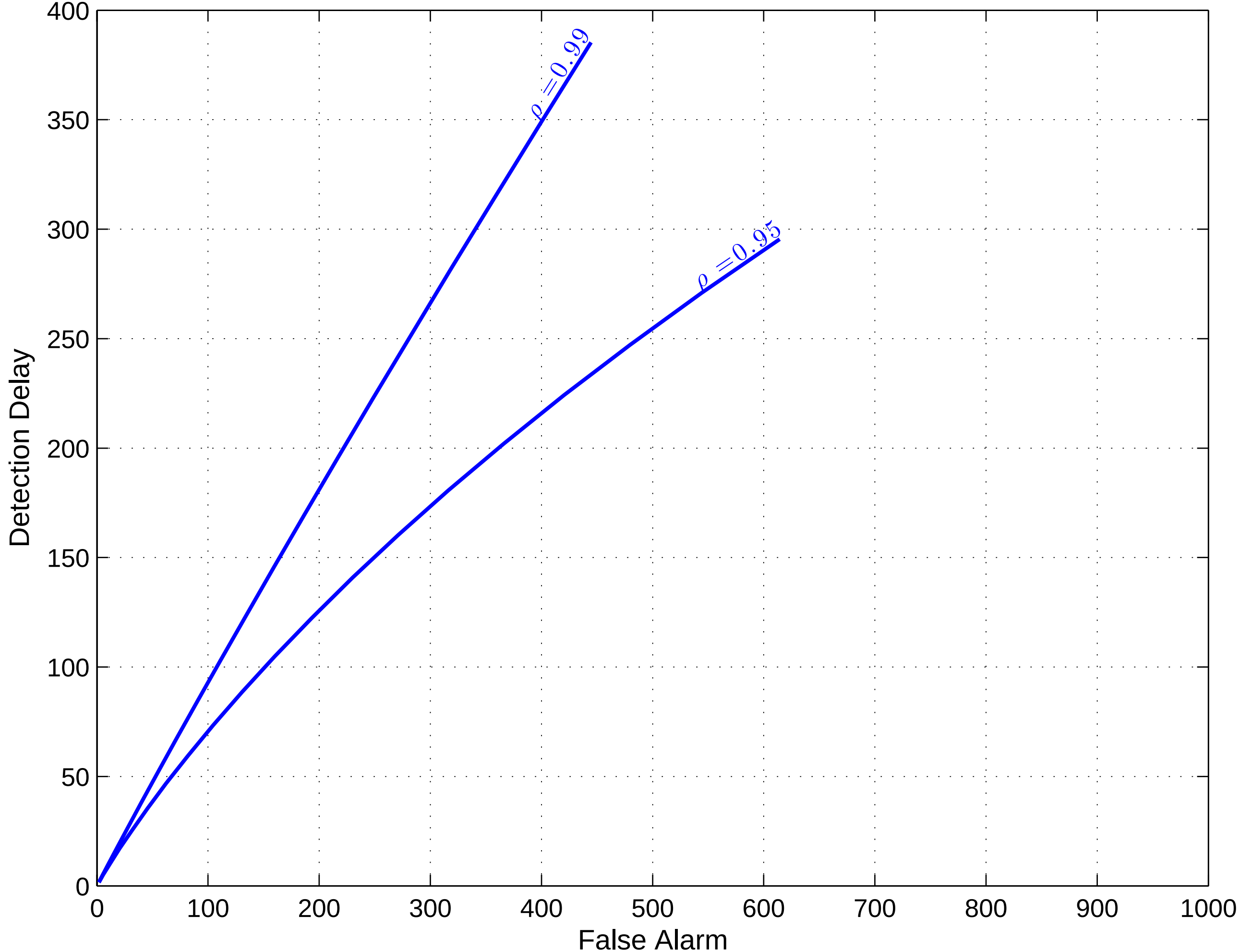
Detection Delay

Impact of Portfolio Size and Age Tranches

	Size	1000			5000			10000		
	Hyp. \ Ages	60-90	60-75	76-90	60-90	60-75	76-90	60-90	60-75	76-90
deaths	100% → 95%	596	710	498	246	99	107	240	99	106
	100% → 90%	244	320	186	106	55	59	112	55	58
	100% → 85%	92	122	100	58	35	36	61	34	36
time	100% → 95%	1086	1130	1120	576	617	422	308	327	212
	100% → 90%	931	1124	947	276	373	241	151	192	127
	100% → 85%	707	980	734	161	247	159	84	127	80



Detection Delay
Detection Delay
False Alarm
ρ =0.85
ρ =0.8
ρ =0.75
ρ =0.7
ρ =1.15
ρ =1.2
ρ =1.25
ρ =1.3
0
50
100
150
200
0
100
200
300
400
500
600
700
800
900
1000



Detection Delay
False Alarm
0
50
100
150
200
250
300
350
400
0
100
200
300
400
500
600
700
800
900
1000
$\rho = 0.99$
$\rho = 0.95$

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